SESSION 4A

PAPER 3

AGATHE TYCHE
OF NERVOUS NETS - THE LUCKY RECKONERS

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by

DR. W. S. McCULLOCH

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SUMMARY

VENN diagrams, with a jot in every space for all cases in which given logical functions are true, picture their truth tables. These symbols serve as arguments in similar expressions that use similar symbols for functions of functions. When jots appear fortuitously with given probabilities or frequencies, the Venn diagram can be written with 1's for fixed jots, 0's for fixed absence, and p's for fortuitous jots. Any function is realizable by many synaptic diagrams of formal neurons of specified threshold, and the fortuitous jots of their symbols can be made to signify a perturbation of threshold in an appropriate synaptic diagram.

Nets of these neurons with common inputs embody hierarchies of functions, each of which can be reduced to input-output functions pictured in their truth tables. The rules of reduction are simple, even for fortuitous jots, and thus formalize probabilistic logic. Minimal nets of neurons are sufficiently redundant to stabilize the logical input-output function despite common shifts of threshold sufficient to alter the function computed by every neuron, and to secure reliable performance of nets of unreliable neurons. Both types of nets are flexible as to the functions they can compute when controlled by imposed changes of threshold.

The neurons, the variations of their thresholds, their excitations and inhibitions are realistic; and there remains sufficient redundancy for statistical control of growth to produce the synapsis of these stable, reliable and flexible nets.

NEUROPHYSIOLOGISTS are indebted to John von Neumann for his studies of components and connections in accounting for the steadiness and the flexibility of behaviour. In speaking to the American Psychiatric Association *(ref. 11)* he stressed the utility and the inadequacy of known mechanisms for stabilizing nervous activity, namely, (a) the threshold of nonlinear components, (b) the negative feedback of reflexive mechanisms, (c) the internal switching to counteract changes - "ultrastability" - *(ref. 1)*, and (d) the redundancy of code and of channel. He suggested that the flexibility might depend upon local shifts of thresholds or incoming signals to components that are more appropriate to computers than any yet invented. His Theory of Games *(ref. 13)* has initiated studies that may disclose several kinds of stability and has indicated where to look for logical stability under common shift of threshold. His "Toward a Probabilistic Logic" *(ref. 12)* states the problem of securing reliable performance from unreliable components, but his solution requires better relays than he could expect in brains. These, his interests, put the questions we propose to answer. His satisfaction with our mechanisms for realizing existential and universal quantification in nets of relays *(refs. 6, 15)* limits our task to the finite calculus of propositions. Its performance has been facilitated by avoiding the opacity of the familiar symbols of logic and the misleading suggestions of multiplication and addition modulo two of the facile boolean notation for an albegra that is really substitutive *(refs. 8, 9, 10)*. Our symbols have proved useful in teaching symbolic logic in psychological and neurological contexts *(ref. 3)*. Familiarity with them undoubtedly contributed to the invention of the circuits whose redundancy permits solution of our problems.

The finite calculus of propositions can be written at great length by repetitions of a stroke signifying the incompatibility of its two arguments. The traditional five symbols, '~' for 'not'; '.' for 'both'; 'v' for 'or'; '⊃' for 'implies'; and '≡' for 'if and only if', shorten the text but require conventions and rearrangements in order to avoid ambiguities. Economy requires one symbol for each of the sixteen logical functions of two propositions. The only necessary convention is then one of position or punctuation.

Since the logical probability and the truth value of a propositional function are determined by its truth table, each symbol should picture its table. When the place in the table is given, any jot serves for "true" and a blank for "false". When the four places in the binary table are indicated by '×' it is convenient to let the place to the left show that the first proposition alone is the case; to the right, the second; above, both; and below, neither. Every function is then pictured by jots for all of those cases in which the function is true. Thus we write A × B for contradiction; A ·× B for A · ~ B; A ×̇ B for A · B; A ×· B for B · ~ A; A ×̣ B for ~ A · ~ B; A ·×̇ B for A · (Bv ~ B); A ×̇· B for (Av ~ A) · B; A ×̣· B for ~ A · (Bv ~ B); A ·×̣ B for (Av ~ A) · ~ B; A ·×· B for (A · ~ B) v (~ A · B); A ×̣̇ B for A ≡ B; A ·×̣̇ B for B ⊃ A; A ·×̇· B for A v B; A ×̣̇· B for A ⊃ B; A ·×̣· B for ~(A · B); and A ·×̣̇· B for tautology. The × or chi, may be regarded as an elliptical form of Venn's diagram for the classes of events of which the propositions A and B are severally and jointly true and false; for in *fig. 1*, the chi remains when the dotted lines are omitted. Similar symbols can therefore be made, from Venn symbols, for functions of more than two arguments. Each additional line must divide every pre-existing area into two parts. Hence, for the number of arguments δ there are 2^{δ} spaces for jots and $2^{2^{\delta}}$ symbols for functions. (*See fig. 1.*)

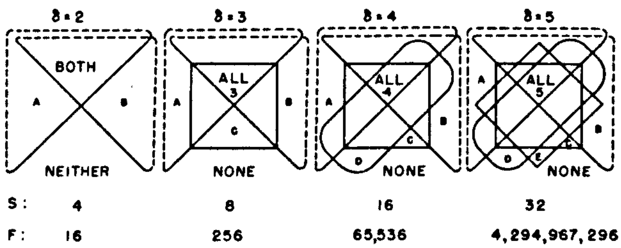

Fig.1. Venn figures with spaces for all intersections of δ classes. S is the number of spaces; F, the number of functions.

Formulas composed of our chiastan symbols are transparent when the first proposition is represented by a letter to the left of the $\times$ and the second to the right. When these spaces are occupied by expressions for logical variables, the formula is that of a propositional function; when they are occupied by expressions for propositions, of a proposition; consequently the formula can occupy the position of an argument in another formula.

Two distinct propositions, A and B, are independent when the truth value of either does not logically affect the truth value of the other. A formula with only one $\times$ whose spaces are occupied by expressions for two independent propositions can never have an $\times$ with no jots or four jots. The truth value of any other proposition is contingent upon the truth values of its arguments. Let us call such a proposition "a significant proposition of the first rank."

A formula of the second rank is made by inserting into the spaces for the arguments of its $\times$ two formulas of the first rank; for example, (A $\cdot\!\times$ B) $\dot{\times}$ (A $\times\!\cdot$ B). When the two propositions of the first rank are composed of the same pair of propositions in the same order, the resulting formula of the second rank can always be equated to a formula of the first rank by putting jots into the $\times$ for the corresponding formula of the first rank according to the following rules of reduction:

Write the equation in the form $(\ldots x_1 \ldots)\ x_2\ (\ldots x_3 \ldots) = (\ldots x_4 \ldots)$; wherein the x_j are chiastan symbols:

(1) If x_2 has a jot on its left, put a jot into x_4 in every space where there is a jot in x_1 and no corresponding jot in x_3. Thus,

$$(A \underset{\cdot}{\times}\!\cdot B) \cdot\!\times (A \underset{\cdot}{\dot{\times}} B) = (A \times\!\cdot B)$$

(2) If x_2 has a jot on its right, put a jot into x_4 in every space where there is a jot in x_3 and no corresponding jot in x_1. Thus,

$$(A \underset{\cdot}{\times}\!\cdot B) \times\!\cdot (A \underset{\cdot}{\dot{\times}} B) = (A \cdot\!\times B)$$

(3) If x_2 has a jot above, put a jot into x_4 in every space where there is a jot in both x_1 and x_3. Thus, $(A \underset{\cdot}{\times}\!\cdot B) \dot{\times} (A \cdot\!\underset{\cdot}{\times} B) = (A \underset{\cdot}{\times} B)$

(4) If x_2 has a jot below, put a jot into x_4 in every space that is empty in both x_1 and x_3. Thus, $(A \underset{\cdot}{\times}\!\cdot B) \underset{\cdot}{\times} (A \cdot\!\underset{\cdot}{\times} B) = (A \dot{\times} B)$

If there is more than one jot in x_2 apply the foregoing rules seriatim until all jots on x_2 have been used. Put no other jots into x_4.

By repetition of the construction we can produce formulas for functions of the third and higher ranks and reduce them step by step to the first rank, thus discovering their truth values.

Since no other formulas are used in this article, the letters A and B are omitted, and positions, left and right, replace parentheses.

In formulas of the first rank the chance addition or omission of a jot produces an erroneous formula and will cause an error only in that case for which the jot is added or omitted, which is one out of the four logically equiprobable cases. With the symbols proposed for functions of three arguments, the error will occur in only one of the eight cases, and, in general, for functions of δ arguments, in one of 2^δ cases. If p_δ is the probability of the erroneous jot and P the probability of error produced, $P = 2^{-\delta} p$. In empirical examples the relative frequency of the case in question as a matter of fact replaces the logical probability.

In formulas for the second rank there are three ×'s. If we relax the requirement of independence of the arguments, A and B, there are then 16^3 possible formulas each of which reduces to a formula of the first rank. Thus the redundancy, R, of these formulas of the second rank is $16^3/16 = 16^2$. For functions of δ arguments, $R = (2^{2^\delta})^\delta$.

To exploit this redundancy so as to increase the reliability of inferences from unreliable symbols, let us realize the formulas in nets of what von Neumann called neurons (3). Each formal neuron is a relay which on receipt of all-or-none signals either emits an all-or-none signal or else does not emit one which it would otherwise have emitted. Signals approaching a neuron from two sources either do not interact, or, as we have shown *(refs. 5,7)*, those from one source prevent some or all of those from the other source from reaching the recipient neuron. The diagrams of the nets of *fig. 2* are merely suggested by the anatomy of the nervous system. They are to be interpreted as follows.

A line terminating upon a neuron shows that it excites it with a value +1 for each termination. A line forming a loop at the top of the neuron

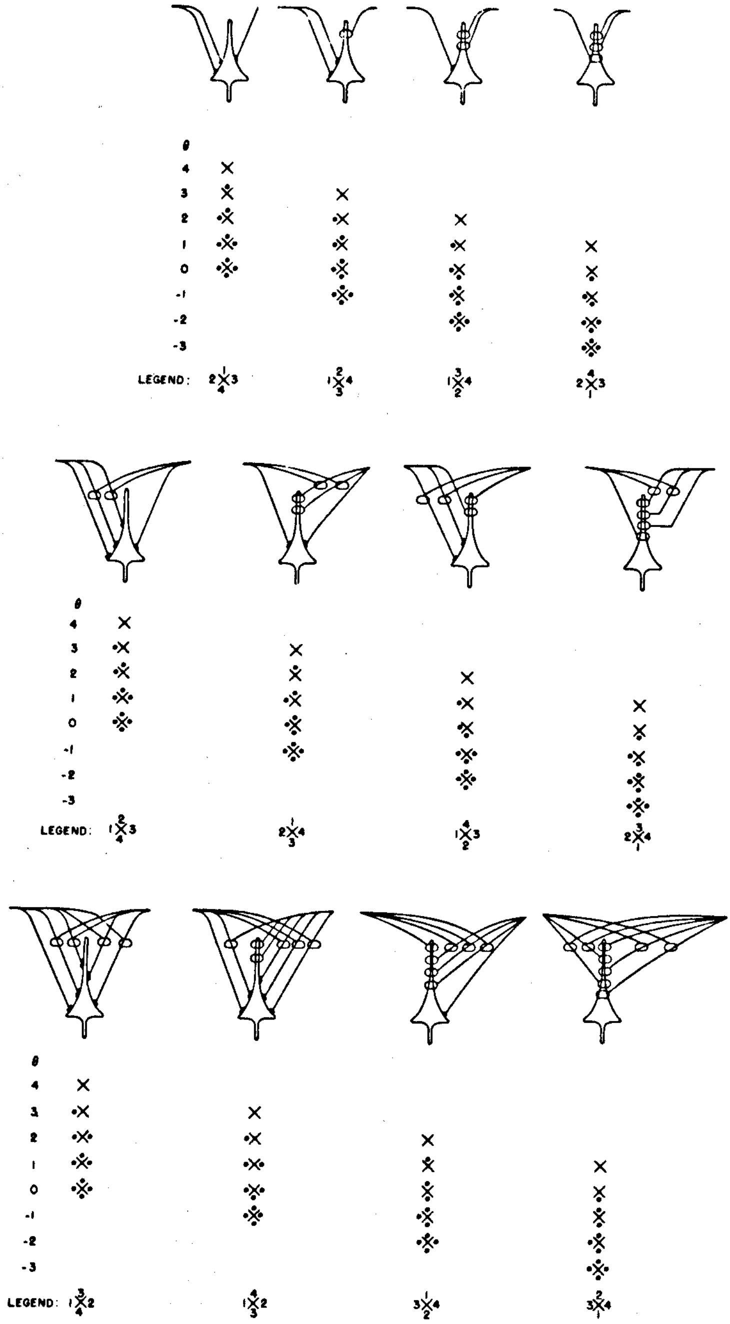

Fig.2. Synaptic diagrams for ·× appearing before ×.

shows that it inhibits it with a value of excitation of -1 for each loop. A line forming a loop around a line approaching a neuron shows that it prevents excitation or inhibition from reaching the neuron through that line.

Each neuron has on any occasion a threshold, θ measured in steps of excitation, and it emits a signal when the excitation it receives is equal to or greater than θ. The output of the neuron is thus some function of its input, and which function it is depends upon both its local connections and the threshold of the neuron. These functions can be symbolized by ×'s and jots beginning with none and adding one at a time as θ decreases until all four have appeared in the sequence noted in the legend for its diagram in *fig. 2.* These are the simplest diagrams fulfilling the requirement. All simpler diagrams are degenerate, since they either fail to add one jot or else add more than one jot for some step in θ. Because all 24 sequences (of which only 12 left-handed are drawn) are thus realized, we can interpret the accidental gain or loss of a jot or jots in an intended × as a change on the threshold of an appropriate neuron.

Any formula of the second rank is realized by a net of three neurons each of whose thresholds is specified; for example, see *fig. 3.* The formula can be reduced to one of the first rank whose × pictures the relation of the output of the net to the input of the net.

When all thresholds shift up or down together, so that each neuron is represented by one more, or one less, jot in its × but the reduced formula is unaltered, the net is called "logically stable."

The redundancy of formulas of the second rank provides us with many examples of pairs of formulas and even triples of formulas that reduce to the same formula of the first rank and that can be made from one another by common addition or omission of one jot in each ×, and the diagrams of *fig. 2* enable us to realize them all in several ways: For example, there are 32 triples of formulas and 64 logically stable nets for every reduced formula with a single jot. Even nets of degenerate diagrams enjoy some logical stability; for example ×̣ ×̣ ·×̣· = ×̇ goes to ·×̣· ·×̣· ·×̣̇· = ×̇..

If such nets are embodied in our brains they answer von Neumann's repeated question of how it is possible to think and to speak correctly after taking enough absinthe or alcohol to alter the threshold of every neuron. The limits are clearly convulsion and coma, for no formula is significant or its net stable under a shift of θ that compels the output neuron to compute tautology or contradiction. The net of *fig. 3* is logically stable over the whole range between these limits. Let the causes and probabilities of such shdfts be what they may, those that occur simultaneously throughout these nets create no errors.

Logically stable nets differ greatly from one another in the number of errors they produce when thresholds shift independently in their neurons and the most reliable make some errors; for example, the net of *fig. 4.*

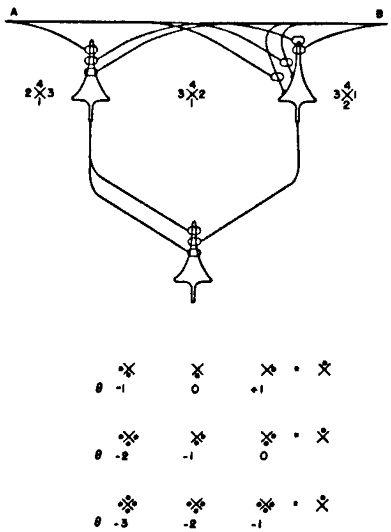

Fig.3. A logically stable net for ẋ..

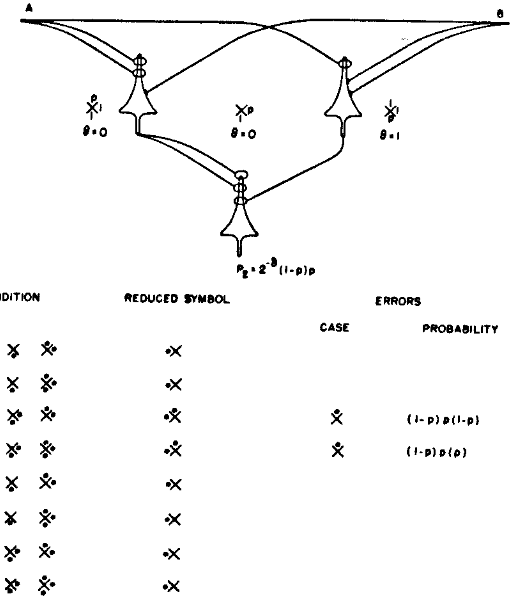

Fig.4. A best stable net for ·×.

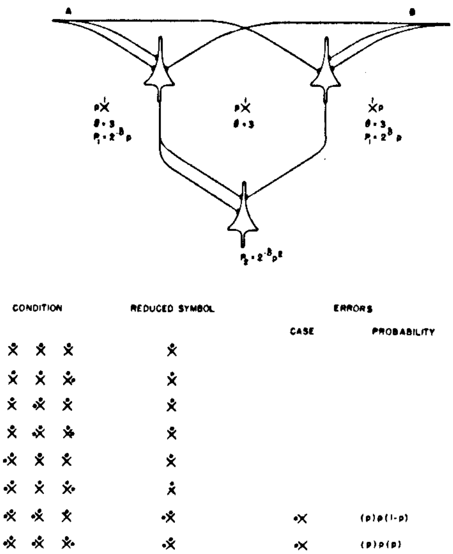

Fig.5. A best unstable net for ẋ..

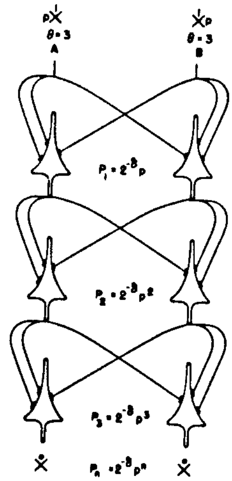

Fig.6. Unstable improvement net for ẋ.

To include independent shifts, let our chiastan symbols be modified by replacing a jot with 1 when the jot is never omitted and with p when that jot occurs with a probability p, and examine the errors extensively as in *fig. 4*. Here we see that the frequency of the erroneous formulas is $p(1-p)$, and the actual error is a deficit of a jot at the left in the reduced formula in each faulty state of the net, i.e., in one case each. Hence we may write for the best of stable nets $P_2 = 2^{-2}\, p(1-p)$. The factors p and $(1-p)$ are to be expected in the errors produced by any net which is logically stable, for the errors are zero when $p = 0$ or $p = 1$. No stable net is more reliable.

No designer of a computing machine would specify a neuron that was wrong more than half of the time; for he would regard it as some other neuron wrong less than half of the time; but in these more useful of logically stable circuits, it makes no difference which way he regards it, for they are symmetrical in p and $1 - p$. At $p = 1/2$, the frequency of error is maximal and is $P_2 = 2^{-2}\, 1/2(1 - 1/2) = 1/16$, which is twice as reliable as its component neurons for which $P_1 = 2^{-2}\, 1/2 = 1/8$.

Among logically unstable circuits the most reliable can be constructed to secure fewer errors than the stable whenever $p < 1/2$. The best are like that of *fig. 5*. The errors here are concentrated in the two least frequent states and in only one of the four cases. Hence $P_2 = 2^{-2}\, p^2$.

Further improvement requires the construction of nets to repeat the improvement of the first net and, for economy, the number of neurons should be a minimum. For functions of δ arguments each neuron has inputs from δ neurons. Hence the width of any rank is δ, except the last, or output, neuron. If n be the number of ranks, then the number of neurons, N, is $\delta(n-1) + 1$.

Figure 6 shows how to construct one of the best possible nets for the unstable ways of securing improvement with two output neurons as inputs for the next rank. The formulas are selected to exclude common errors in the output neurons on any occasion. In these, the best of unstable nets, the errors of the output neurons are $P_n = 2^{-\delta} p^n$.

[Whether we are interested in shifts of threshold or in noisy signals, it is proper to ask what improvement is to be expected when two or three extra jots appear in the symbols. With our nondegenerate diagrams for neurons a second extra jot appears only if the first has appeared, and a third only if the second. If the probability p of an extra jot is kept constant, the probability of two extra jots is p^2 and of three is p^3. Examination of the net in *fig. 6* shows that $P_2 < P_1$, if $p + p^2 + p^3 < 0.15$ or $p < 0.13$. To match Gaussian noise the log of successive p's should decrease as $(\Delta\theta)^2$, or 1, p, p^4, p^9 giving $P_2 < P_1$ for $p < 0.25$. The remaining errors are always so scattered as to preclude further improvement in any subsequent rank.]

When common shifts of θ are to be expected, or all we know is $0 < p < 1$, a greater improvement is obtained by alternating stable and unstable nets as in *fig.7*, selected to exclude common errors in its output neurons. For n even

$$P_n = 2^{-\delta} p^{n/2} (1-p)^{n/2}$$

and the expected error is

$$2^{-\delta} \int_0^1 p^{n/2} (1-p)^{n/2} \, dp = 2^{-\delta} \frac{\left(\frac{n}{2}!\right)^2}{(n+1)!}$$

which is less than with any other compositions of $\delta = 2$ nondegenerate diagrams.

When $\delta = 3$, the redundancy, $R = \left(2^{2^\delta}\right)^\delta$, provides so many more best stable and best unstable nets that the numbers become unwieldy. There are $\left(2^{2^\delta}\right)^{\delta+1}$ nets for functions of the second rank each made of 4 neurons to be selected from 8! diagrams with 9 thresholds apiece. Formerly *(ref. 4)* I said it was clear that the best stable and unstable nets for $\delta < 2$ are better than those for $\delta = 2$ only in the factor $2^{-\delta}$ for error in a single case. That is only true if the nets are composed of nondegenerate diagrams alone. With neurons $\delta = 3$, a single degenerate diagram for the output neuron permits the construction of a logically stable net with $P_2 = 0$, even with independent shifts of θ sufficient to alter the logical function computed by every neuron, as seen in *fig.8*. The same degenerate diagram for the $\delta = 3$ output neuron receiving inputs from three nondegenerate $\delta = 2$ neurons, selected to make but one error in each case, is likewise stable and has an error-free output despite independent shifts of θ, as is seen in *fig.9*.

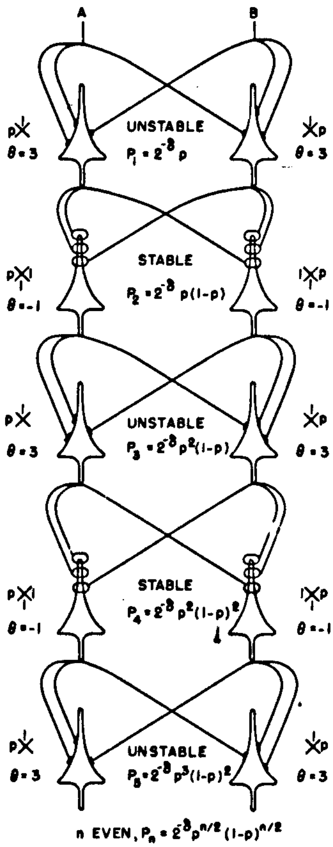

Fig.7. Alternating improvement net for ×..

None of these nets increases reliability in successive ranks under von Neumann's condition that neurons fire or fail with probability p regardless of input; but they are more interesting neurons. They are also more realistic. In the best controlled experiments the record of a single unit, be it cell body or axon, always indicates firing and failing to fire at

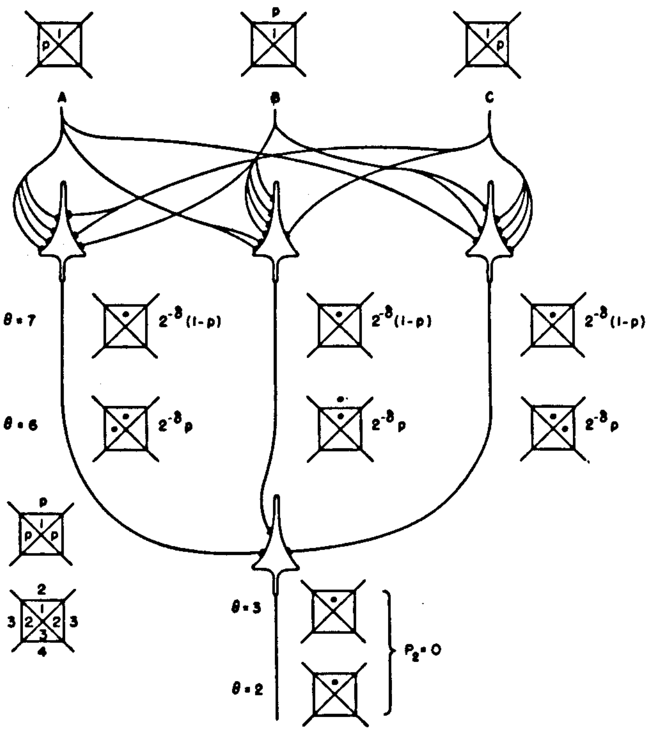

Fig.8. Input $\delta = 3$, output degenerate $\delta = 3$ neuron for $\dot{x}$.

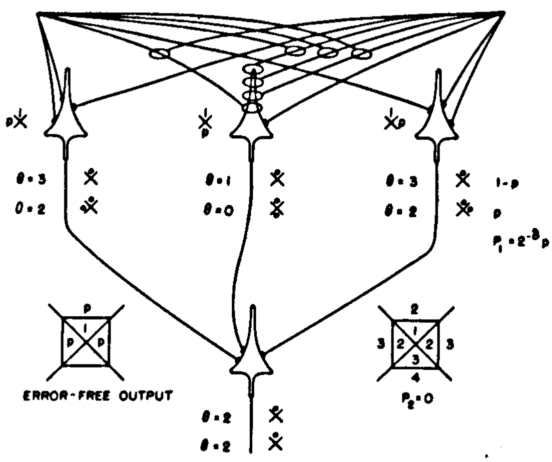

Fig.9. Input $\delta = 2$, output degenerate $\delta = 3$ neuron for $\dot{x}$.

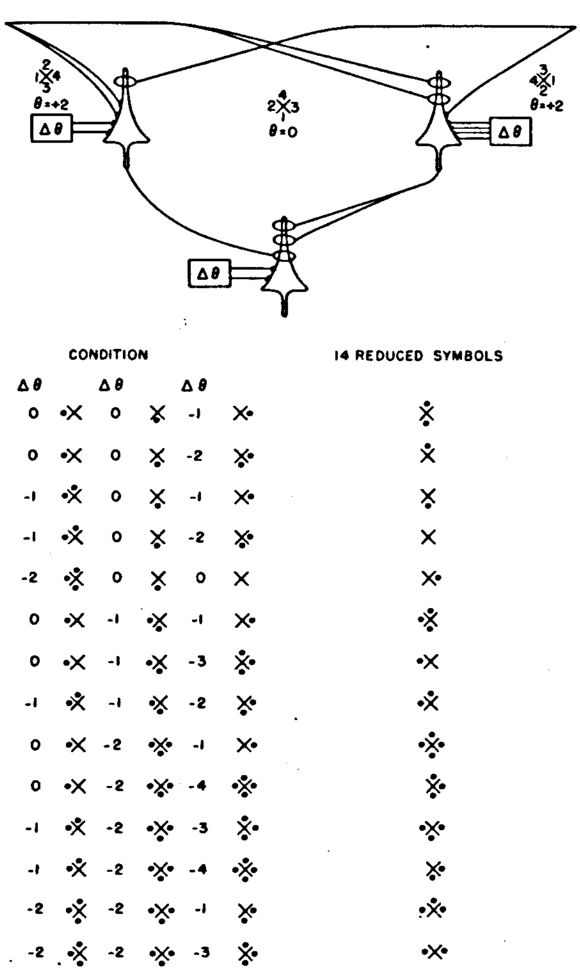

Fig.10. Flexible net, always unstable.

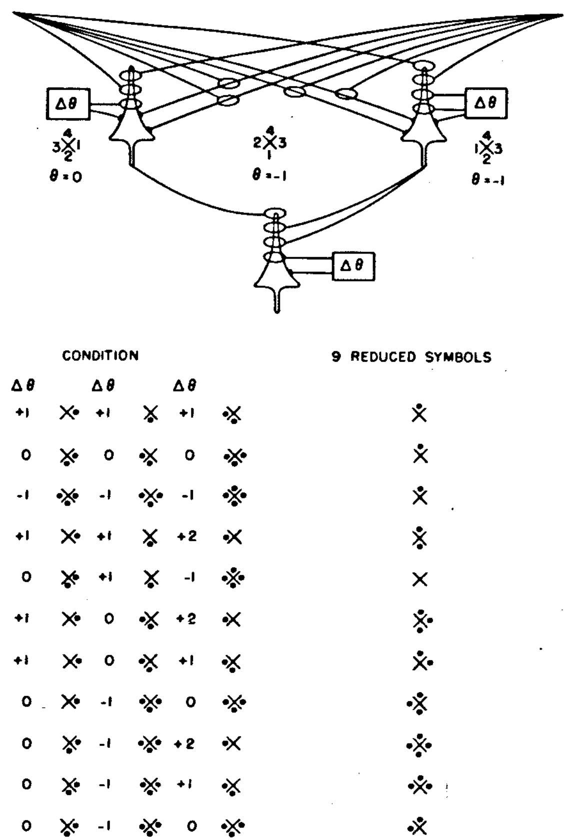

Fig.11. Flexible net, logically stable for ẋ.

near threshold values of constant excitation, whether it is excited transynaptically or by current locally applied. At present, we do not know how much of the observed flutter of threshold is due to activity of other afferent impulses and how much is intrinsic fluctuation at the trigger point. We have not yet a reasonable guess as to its magnitude in neurons in situ, and for excised nerve we have only two estimates of the range: one, about 2 per cent; and the other, 7 per cent *(refs. 2, 14)*. Our own measurements on a single node of Ranvier would indicate a range of 10 per cent. To discover regularities of nervous activity we have naturally avoided stimulation near threshold. Now we must measure the intrinsic jitter, for this determines both the necessity of improving circuits and the maximum number of afferent terminals that can be discriminated. Eventually we will have to take into account the temporal course of excitation and previous response, for every impulse leaves a wake of changing threshold along its course.

Despite the increase in reliability of a net for a function of the second rank some of these nets can be made to compute as many as 14 of the 16 reduced formulas by altering the thresholds of the neurons by means of signals from other parts of the net, as in *fig. 10*, and even some logically stable nets for triples of formulas can be made to realise 9 of the 16, as in *fig. 11*. Even this does not exhaust the redundancy of these nets, for both of them can be made to compute many of these functions in several ways.

The diagrams of *fig. 2* were drawn to ensure a change in function for every step in θ. Actual neurons have more redundant connexions. We are examining how to use this redundancy to design reliable nets the details of whose connexions are subject to statistical constraints alone. This is important because our genes cannot carry enough information to specify synapsis precisely.

For the moment, it is enough that these appropriate, formal neurons have demonstrated logical stability under common shift of threshold and have secured reliable performance by nets of unreliable components without loss of flexibility.

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DISCUSSION ON THE PAPER BY DR. W. S. McCULLOCH

MR. E. A. NEWMAN: From the point of view of a computer designer dealing with logical circuits, the ideas expressed in Dr. McCulloch's paper seemed to me brilliant in their inherent simplicity and great power. If one uses a number of two-input devices together, as Dr. McCulloch does, and wants the overall result of one two-input device, then evidently the redundancy is very great, and the overall system can be chosen in a way that gives great stability.

Any complicated data-processing system must store and handle a hierarchy of ideas. Dr. McCulloch's scheme enables such a hierarchy to be handled in a way that gives little or no redundancy for the mass of non-vital detail, and hence maximum overall storage efficiency, and at the same time great redundancy for those basic patterns which are absolutely vital.

DR. GREY WALTER: I should like to comment on the notion that a high degree of trustworthiness is essential and attainable for a nervous system, even when its neurons are liable to failure. In Dr. McCulloch's schemata we see that this can be so but there are interesting limits and exceptions. The story is, I believe, that some of these ideas occurred to Dr. McCulloch in a conversation with von Neumann about the latter's ability to drive his car when he had been drinking. To me it seems equally surprising that small quantities of alcohol and other drugs do actually affect behaviour rather dramatically - in concentrations that is, that do not affect the individual nerve cells appreciably. This vulnerability to general assault by infiltration has to be accounted for as well as the apparent indifference of the nervous system to the fate of individual neurons.

One can conceive of the input to a nervous system as consisting, among other things, of a chain or cascade of neurons. Now if the passage of an impulse from one end to the other is a question of probability, then in the ideal normal case when transmission is perfect, the probability of an impulse which enters this chain from the periphery getting to some part of the nervous system is unity. In physiological terms this means that the impulse from each neuron is of supraliminal intensity at the synapse with each succeeding neuron. Now if some general influence - such as alcohol - is brought to bear so that the probability of transmission for each synapse is slightly reduced, then the probability of transmission along the whole chain will be diminished by a greater factor, depending on the number of links or synapses; the cascade acts as a probability amplifier, or more precisely attenuator since the value of the probability will usually be fractional. For example, if there are ten neurons in the cascade and the general influence, von Neumann's martinis or a sleepless night or whatever, reduces the impulse passage probability from unity to 0.5 for each synapse, then the probability of an impulse getting right through the ten links will be reduced to $(1/2)^{10}$, and a chance of a thousand to one against means not merely collision but coma. A series chain of elements is therefore very vulnerable and furthermore when it makes up part of a network, the system is likely to be anisotropic - its lengthways probability conductance can easily become quite different from its sideways conductance, which in the example given will still be 0.5 for impulses crossing the chain through a single synapse.

In the type of neuron circuitry schematised by Dr. McCulloch, this might be a valuable aid to understanding the specific effects of general influences such as drugs, hormones and the like. It has been shown that most of the psychotropic compounds exert their action mainly by way of the complex neuron networks in the base of the brain. These reticular formations control the inputs and outputs of the brain and are known to be particularly vulnerable also to mechanical disturbance in spite of their situation. If we consider these systems in the terms suggested by Dr. McCulloch we can see that his schemata allow for individual component failure but expose the system to interference by general influences. This can account for the variety and intricacy of the homeostatic mechanisms and structural safeguards that support the brain as a whole, while the fate of the individual neuron is left to chance. The analogies with computer design and with social systems are obvious.

DR. W. K. TAYLOR: Dr.McCulloch recognises many of the limitations of his model neurones but I should like to know whether he has considered one point about them that particularly worries me and which I can demonstrate with a simple example of two inputs and a threshold of 2. If both inputs consist of 1 millisecond duration pulses at a steady rate of 200 per second, the unit will give 200 pulses per second out if the inputs are in step but no output at all if the pulses are more than one millisecond out of step, since they will then never summate to reach the threshold of 2. We know that there is no accurate timing device in the nervous system, as there is in a digital computer, and that variable delays can occur in nerve fibres and at synapses. If the delay is more than the pulse width at one input of the McCulloch model neuron unit the output falls from 200 pulses/sec. to zero in the example given and I suggest that the unit would be a particularly unlucky reckoner.

DR. W. S. McCULLOCH: I should like to answer those two separately first. In the first place, the dissimilarity of properties from neuron to neuron, where we happen to know them, is very, very, great. Patrick Wall has been working, during the last year, on the afferents to the spinal cord, and the first relays that ascend in the dorsolateral portion of the spinal cord presumably destined for the cerebellum. They can be excited, some by stretch receptors and some by touching the skin. If you map the area that any one axon brings into the spinal cord, you get a reasonable area. It is small. If you map the area of the skin covered by any one of these cells whose axons go up towards the cerebellum - this is the skin I am talking about - you find it is about 25 times as great. Now comes the curious thing. You can take that animal and soak him in barbiturates and you do not get a change in the area. You can strychninise him till he is ready to jump off the table, and you do not get a change in the area. Exactly the reverse is true, of course, of the motor neuron. Here you have two neurons, each of which has direct connections from the periphery but they are entirely different in their behaviour: one has no subliminal fringe that we can discover, the other certainly has. Now I am well aware that the behaviour of these cells is also quite different in response to an input. A motor neuron under these circumstances, when you give it a boomp coming in - the typical behaviour of these cells is to put out a whole series of impulses when they receive one; and what you change when you drug that animal is you increase the length of the barrage with strychnine and you decrease it with barbiturates, but as long as it will respond at all it will respond to the whole area - quite a different organisation from that of motor neurons.

Now my neurons are very far removed from a good Hodgkin-Huxley neuron. The reason is perfectly obvious. I only want to embody in these neurons a certain logical aspect of a theory. That is all that I propose to do. I do not think they are particularly realistic. I think they are more realistic than the Eccles-Jordan proposals of mere flip-flops for neurons, in that they do introduce a notion of threshold; and I think it is out of this notion of threshold that the power of this kind of logic comes.

Fundamentally, we in biology are famous for our inability to handle mathematics. That is certainly true of us, by and large; but the other thing is this, that almost all mathematics was made for the physicist and not for the biologist. We do not generally have the mathematics we need, and we go on to invent it for ourselves, playing with spools of string and bits of rubber. What I wanted with my map was to get the rubber in the right place. I wanted not $f(x)$ where x was probable but where the f was probable. That is all that those normal neurons are supposed to do.

The next thing I want to say pertains to the second question. The second question is of this kind: what a neuron will do under these circumstances depends on the threshold of the neuron. If the threshold of the neuron is 2 and there are two afferents to it, it is a coincidence detector, and if we say one afferent has a rate A and the other has a rate B, the output will be some constant times the product of A and B (these are frequencies) - it becomes a product-taker. All coincidence detectors will do this. If the value of the threshold was 1, then you will get the simple sum, so until you have specified thresholds of these kind of components, you do not know what they are going to do.

DR. A. M. UTTLEY: And being a product taker, it will be a very lucky reckoner if you want cross-correlation?

DR. W. S. McCULLOCH: That is right.

DR. F. ROSENBLATT: I would like to add just a couple of comments to Dr. McCulloch's defence of his neurone. First of all, we have made several observations on the effect of changing thresholds in statistically connected networks in connexion with our work on the perceptron; and one of the interesting things is that it makes very little difference in such networks what the threshold is, within rather wide limits, provided the threshold is fairly high. Given a threshold sufficiently greater than zero, then we can practically double this threshold and a network which is made up of essentially random connections still learns in about the same way, with a greater or a lesser efficiency. It is also possible to change the properties of the neurones in rather drastic ways without seriously altering their performance. For example, if we substitute the model which I proposed in my own paper the continuous transducer neurone which responds on a frequency basis, in place of one which responds at fixed time increments - which I think really takes care of Dr. Taylor's problem rather nicely in most cases - it turns out that there is a negligible difference between such neurones and Dr. McCulloch's neurone. As a matter of fact, the difference is so negligible that it has proved to be no longer worthwhile to maintain this distinction in our own work because we now find we can analyse these circuits much more satisfactorily using something much more like a conventional McCulloch-Pitts neurone and come out with almost identical numerical results. The difference between a continuous transducer neurone and a neurone which responds at fixed time increments makes very little difference. However, some non-zero threshold is essential; as soon as the threshold falls to zero the behaviour of the system goes to pot and we get essentially no learning out of it at all.

DR. M. L. MINSKY: I do not quite follow one of Dr. Walter's remarks. It seems to me that if, under the influence of different drugs, you can find grossly different local neural behaviors and yet the same functional behavior, this is evidence that you do have some sort of mechanism of the kind Dr. McCulloch suggests, in which the properties of neurones can vary without changing the properties of larger sub-nets.

In connection with Dr. Rosenblatt's remark, I do not see a direct connection between that kind of reliability and this. If you have a machine in which the initial properties of the neurons do not critically affect the learning behavior, then indeed early changes in threshold should not have much effect. But it does *not* follow from that alone that after the machine has become organized you can go ahead and change thresholds grossly and expect it to maintain the same behavior. I would like to ask him if there is evidence for a high degree of this kind of stability in a "Perceptron" that has learned.

DR. F. ROSENBLATT: Most of our evidence does concern initial changes in threshold; this is quite true. However, we also have some evidence that we can change the threshold (within reasonable limits) *after* the system has learnt, and the performance will not be too seriously affected. There will be some deficit: if we lower the threshold we are allowing additional cells to respond, which would not otherwise have responded and this does correspond to some introduction of noise into the system. However, the statistical bias which serves as our 'memory' phenomenon tends to be retained in spite of the introduction of additional noise due to the changes in threshold. This does not have quite the type of stability Dr. McCulloch proposes, it is true. However, we do have here a system which is particularly insensitive to threshold changes, even after learning.

DR. M. B. BARLOW: There is one small point - Dr. McCulloch is a physiologist, and I think it would be rather dreadful if everybody thought that all physiologists would agree with a model neuron like that. I wonder whether he would stand up and say that he does not believe real neurons are as simple as the neurons he is talking about.

DR. W. S. McCULLOCH: I thought I had made that very clear. I have no notion that these neurons of mine are anything more than an embodiment of arithmetic. I know that a real neuron has an 8th order non-linear differential equation, which I cannot on inspection handle at all. That is not the kind of device with which to explain a logical problem when what you are really trying to do is set up a probabilistic logic.

PROF. J. Z. YOUNG, CHAIRMAN: Would it be better not to use the word "neuron" then?

DR. W. S. McCULLOCH: I am sorry. I call them formal neurons because this is the word von Neumann had used and I followed him through on it, thats all. The great point is they are not to be confused with ordinary relays that merely close or open a circuit. Everyone says, "Oh, is this not the same as Shannon's Hammock net?" No, it is not. Shannon and I have been over this problem. It is an entirely different problem. His is one of maintaining or obtaining continuity through a system, and its solution is entirely different from mine. They do not even map on each other.

PROF. Y. BAR-HILLEL: I should like to call Dr. McCulloch's attention to other existing pictorial representations of the truth-functional connectives, especially to the diagonal symbolism of Charles S. Peirce (of some 80 years standing), the wheel symbolism of the late Polish logician Stefan Lesniewski, and the recent trapezoid symbolism of W. T. Parry. Some of these symbolisms might perhaps be slightly more convenient for Dr. McCulloch's purposes than his own. Compare, e.g., the following three notations for material implication, if A then B, the customary Russelian symbolization of which is A B:

McCulloch	Peirce	Lesniewski	Parry
A ·✕· B	A (X B	A ϕ– B	A ⊃ B

You will notice that they are all based on the same principle, i.e. the exploitation of the Boolean expansion (the developed disjunctive normal form).

For all this, see the two papers by W. T. Parry, *(ref. 1)* and G. B. Standley *(ref. 2)*. The second paper describes an ideographical method of computation with Parry's trapezoidal symbols which is again closely reminiscent of Dr. McCulloch's procedure.

MR. E. A. NEWMAN: I would refer to the comments of Dr. Taylor and Dr. Barlow.

As an engineer it seemed to me evident that when Dr. McCulloch refers to, say, an "and" element with two inputs and one output, he is referring to a generallised element capable of performing the "and" operation. He could, for example, quite well be referring to a device in which the two inputs took the form of pulse trains having a frequency proportional to the logarithm of the drive, and the output a pulse train having the sum frequency - or in fact to any of the hundreds of devices with the same logical implications. I am very surprised therefore that physiologists should think that, when Dr. McCulloch adopts a simple logical notation for the functional behaviour of his neurons, he should be implying that they obtain their functional behaviour in precisely the way directly implied by the diagrams.

REFERENCES

1. PARRY, W. T. A new symbolism for the propositional calculus. *Journal of Symbolic Logic*, 1954, **19**, 161.
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DR. M. B. BARLOW: I was prompted to make my remarks by the fact that engineers and others have, in the past assumed rather too simple properties for neurons. As long as they all stop doing so, I am very happy.

DR. W. S. McCULLOCH: I should like to make one remark, and I should like to address a question to your chairman. We expected, from what we thought to be the dimensions of a node of Ranvier along an axon that we should necessarily run into a jitter of thresholds of less than one per cent. On making the measurements under ideal conditions, the actual jitter that we encounter is of the order of ten per cent. Are we over-estimating the area of the trigger point?

PROF. J. Z. YOUNG, CHAIRMAN: I should think very likely. It is a technical point, but the actual available membrane surface there is extremely difficult to compute. By electron-microscopy it looks very different from by light microscopy because there are folds of the Schwann cells all over, and it may be many times less than one would suppose from the actual gap seen in the light microscope.

DR. W. S. McCULLOCH: You see, it is this jitter of threshold which compels me to look for a probabilistic way of handling it, because if that threshold is jittering - the function that the cell is going to compute, if that's its striking point is certainly going to be shifting; and unless I'm prepared to take that into consideration I think my imitations of real neurons in words or in chalk are very unrealistic.

PROF. J. Z. YOUNG, CHAIRMAN: It is a very striking thing if there is that threshold change. Is there other evidence of that?

DR. McCULLOCH: The other evidence goes way back to Blair and Erlanger in those early papers. They have about 20 or 30 nodes of Ranvier in series. They came up with values of the order of 3 per cent.